Course: Semi-Riemannian geometry

Teacher(s): Emilija Nešović, Ljubica Velimirović

Course status: elective

ECTS: 12

Prerequisites: -

Goal

Considering smooth manifolds equipped with metric tensor of arbitrary signature. Studding smooth manifolds with non-degenerate metric tensor by applying tensor calculus.

Goal The students are successfully achieved theoretical knowledge related with smooth manifolds equipped with metric tensor of arbitrary signature and will be able to apply fundamental methods in studying semi-Riemannian manifolds in further research work.

Contents

Theoretical lectures:

Tensors. The notion of the tensor field. Tensor types. Value of the tensor at the point on a manifold. Tensor components. Contraction. Covariant derivative. Tensor Derivation. Symmetric bilinear forms. Index of symmetric bilinear form. Scalar product on vector space.

Semi-Riemannian manifolds. The notion of metric tensor on smooth manifold**.** Definition of semi-Riemannian manifold. Causal character of vectors. Isometries. Levi-Civita connection. Geodesic lines. Exponential map. Riemannian curvature tensor of semi-Riemannian manifold. Sectional curvature of semi-Riemannian manifold. Semi-Riemannian surfaces. Type-changing and metric contraction. Frame fields. Some differential operators. Ricci and scalar curvature of semi-Riemannian manifolds. Semi-Riemannian product manifolds. Local isometries of semi-Riemannian manifolds.

Semi-Riemannian submanifolds. Tangent and normal vector fields. Induced connection on semi-Riemannian submanifold. Geodesics on semi-Riemannian submanifolds. Totally geodesic semi-Riemannian submanifolds. Semi-Riemannian hypersurfaces. Hyperquadrics. The Codazzi equation. Totally umbilic hypersurfaces. Normal connection on semi-Riemannian submanifold. Congruence theorem. Isometric immersion as semi-Riemannian submanifold.

Practical lectures:

Implementation of the theoretically analysed methods.

References:

1. B. O'Neill: *Semi-Riemannian Geometry*, Academic Press, New York, 1983.
2. S.C. Newman, Semi-Riemannian geometry: The Mathematical Language of General Relativity, John Willey & Sons, 2019.
3. B.Y.Chen, Pseudo-Riemannian geometry, delta-invariants and applications, Wold Scientific, Singapore, 2011.

Number of classes per week	Theoretical: 5	Practical:

Methods of teaching

Theoretical lectures and independent work of students during practical hours.

Knowledge estimation: **(max 100 points)**

50 points on pre-exam and 50 points on oral exam