Course: Semigroup Theory

Course instructors: Igor Dolinka, Miroslav Ćirić

Course type: elective

Credit points ECTS: 10

Prerequisites: none

Course objectives:

Introduction to fundamental ideas, concepts and results of semigroup theory, as well as applications of semigroups, primarily in other branches of algebra, mathematical logic, and theoretical computer science.

Learning outcomes:

Upon completing the course, the student should absorb the basic ideas, concepts and results of semigroup theory. The main goal is to arrive at the operative level of knowledge of the theory, whereupon one can apply it in scientific research within a wide range of mathematical areas.

Course description (outline):

Basic notions of semigroup theory

Congruences. Rees congruences and ideals. Ideal extensions. Examples of semigroups: free semigroups, transformation semigroups. Presentations of semigroups. Green's relations. The structure of *D*-classes. The Schützenberger group of a *D*-class. Regular *D*-classes.

Regular semigroups

Simple and 0-simple semigroups. Principal factors. Completely simple and 0-simple semigroups,the Rees-Suschkevitch theorem. Congruences on completely 0-simple semigroups. Completely regular semigroups (unions of groups). Semilattices of groups. Bands, free bands.

Introduction to the theory of semigroup decompositions and compositions

Semilattice decompositions. Band decompositions. Decompositions of semigroups with 0. Subdirect decompositions. Archimedean semigroups and their semilattices. Compositions of semigroups.

Introduction to the theory of inverse semigroups

The natural order of inverse semigroups. Congruences of inverse semigroups. Munn's construction. Simple and bi-simple inverse semigroups. *E*-unitary inverse semigroups and McAlister's *P*-theorem. *E*-unitary covers. Factorisability in inverse semigroups. Free inverse monoids.

Depending on the special interests of students, there is an option for the course to include some current research areas of semigroups theory such as: the theory of ordered semigroups and monoids, combinatorial semigroups theory, the theory of diagram monoids, varieties of semigroups and finite basis problems, pseudovarieties of finite semigroups with applications to automata and formal languages, etc.

References:

1. J.M.Howie, *Fundamentals of Semigroup Theory,* Oxford University Press, New York, 1995.
2. A.H.Clifford, G.B.Preston, *The Algebraic Theory of Semigroups*, American Mathematical Society, Vol. 1, 1961, Vol.2, 1967.
3. M.Petrich, *Introduction to Semigroups*, Merrill Publishing Company, Columbus, Ohio, 1973.
4. J.Rhodes, B Steinberg, *The q-theory of Finite Semigroups*, Springer, New York, 2009.
5. M.Petrich, N.R.Reilly, *Completely Regular Semigroups*, Wiley-Interscience Publication, 1999.
6. S.Bogdanović, M.Ćirić, *Polugrupe*, Prosveta, Niš, 1993.

Active teaching hours: 5	Theoretical classes: 5	Practice classes:

Methods of teaching:

The lectures use classical teaching methods, aided by contemporary information and communication technology and interaction with students. Students' progress during the course is monitored by homework assignments and by means of writing and defending seminar papers. The goal of the final oral exam is to test the comprehensive understanding of the material of the course.

Grading structure (100 points)

Homework and seminars: 30 points. Final oral exam: 70 points.